If y = -x, why are the results for y not the negative of those for x?

Here x and y are percentage change variables in a linear equation in a TAB file.

Modellers are often puzzled as to why simulation results for y are not the negative of those for x. For example, if you solve the underlying model accurately, if x increases by 10% you will find that y only decreases by about 9.09%. Below we explain why.

1 Think About the Underlying Levels Equation

It is helpful to think about what the underlying levels equation must be since accurate simulations via GEMPACK solve the underlying levels equations accurately.

As we show in section 1.1 below, the linear equation

$$y = -x$$

comes from the levels equation

$$Y = 1/X$$

Or, more accurately, from an equation $Y = C/X$ where C is any constant (for example, $Y = 23/X$).

Once you focus on the levels equation $Y = 1/X$, it is easy to see why an accurate result for y is not the negative of the result for x, as the numerical example in section 2 makes clear.

1.1 Linearising Y = 1/X

As usual, linearising an equation by hand is done by differentiation – see section 9.2 of the Release 8 version of GEMPACK document GPD-2.

When doing this, we use c_X to indicate the change in X and p_X to indicate the associated percentage change in X. Of course these are related via

$$p\_X = 100*c\_X/X$$

So let's linearise $Y = 1/X$. Differentiating both sides gives

$$c\_Y = [-1/X*X]*c\_X = -[c\_X/X] * [1/X] = -[c\_X/X] * Y$$

since $Y = 1/X$. Hence we see that

$$c\_Y/Y = -c\_X/X$$

Multiplying both sides by 100 gives

$$100*c\_Y/Y = -100*c\_X/X$$

and hence (remembering how p_X is related to c_X) we see that p_Y = –p_X

as required.

Thus the linearised version of the levels equatino Y = 1/X is

$$y = -x$$

in the usual notation where lowercase letters y and x denote percent changes in Y and X respectively.

1.1.1 Linearising Y = C/X

You might like to repeat the above for this equation where C is a constant. You will find that again the linearised equation is y = –x.

2 Numerical Example – 20% Increase in X

Consider the levels equation Y = 1/X and suppose that, initially, X and Y both start with the value 1. Consider a 20% exogenous increase in X. Then X will move from 1.0 to 1.2 and hence Y will move from 1 to 1/1.2=0.833333. Hence the resulting change c_Y in Y is –0.1666667 and so the percentage change y in Y is –16.66667, which is not the same as –20.

2.1 2-Step Euler Calculation

It is instructive to follow through a 2-step Euler calculation as would be done by GEMPACK for the above example. The linear equation is y = -x and both X and Y start from the value 1.

In the first step, half of the shock is applied to X. X is to change from 1.0 to 1.2 over the whole simulation. So the change in X in the first step is just half of that – namely an increase of 10% (or 0.1) so X moves from 1.0 to 1.1 in this step. What happens to Y? It begins at 1.0 and the equation y = -x says that y (the percent change in Y) in this step will be equal to –10. So Y decreases by 10% which means that it goes from 1.0 to 0.9 in this step.

In the second step, X increases from 1.1 to 1.2, which is an increase of 100*0.1/1.1=9.0909%. Hence y (the percent change in Y) will be –9.0909. That means that Y decreases from 0.9 to

0.9*[1 – 9.0909/100] = 0.8181.

Hence, over the whole 2-step Euler calculation, X increases by 20% (exogenously) while Y decreases from 1.0 to 0.8181 which is a decrease of only 100*0.1819/1=18.19%. So you see that the 2-step Euler result for y is not the negative of that for x.

It is instructive to note that

- increasing X by 20% in a 2-step calculation does not increase X by 5% in the second step, but by slightly less than 5%. That is because a 5% increase followed by a 9.0909% increase compounds to exactly a 10% increase.
- decreasing Y by 10% in the first step and then by 9.0909% in the second step does not result in an overall 19.0909% decrease in Y but slightly less, namely a decrease of 18.19%.

In section 2.3 below we show you a TAB file and associated Command file. You can run with these to reproduce the numerical results above.

2.2 Compounding is the Answer to the Puzzle

Summary.

1. Note that in the 2-step calculation above, the percent change y for Y in each step is the negative of the percent change x for X in each step (as you would expect).
2. The **key to the puzzle** is the **compounding of results**: the compound effect of –10% then –9.0909% (for y) is not the negative of the compounding effect of +10% and +9.0909% (for x).

Another way of looking at compounding of results is to consider fractional changes (the ratio of the change to the original value – these are one hundredth of the percentage change). [In the 2-step example above, the fractional change in x in the first step is f1=0.1 while, in the second step, it is f2=0.0909.]

In general, if f1 and f2 are positive fractional changes,

the compound effect of +f1 and +f2 is f1+f2+f1*f2
since (1+f1)*(1+f2)=1+f1+f2+f1*f2. This is MORE than f1+f2.

the compound effect of –f1 and –f2 is –[f1+f2–f1*f2]
since (1–f1)*(1–f2)=1–f1–f2+f1*f2. The number here f1+f2–f1*f2 is LESS than f1+f2.

Note also that for a 1-step calculation, the y result will be the negative of the x result. [There is no opportunity for compounding to get into the act.]

2.3 TAB and Command File for this Example

Consider the TAB file RECIP.TAB and the associated Command file RECIP.CMF shown below. You might like to type in and save these files on your computer.

```
!   RECIP.TAB   !
Coefficient X_L # Levels value of X # ;
Coefficient Y_L # Levels value of Y # ;
Formula (initial) X_L = 1 ;
Formula (initial) Y_L = 1 ;
Variable  x  # percent change in X # ;
Variable  y  # percent change in Y # ;
Update X_L = x ;    Update Y_L = y ;
Write X_L to terminal ; Write Y_L to terminal ;
Equation E_y       y = -x ;
Write (postsim) X_L to terminal ;
Write (postsim) Y_L to terminal ;
```

```
!  RECIP.CMF
dws = yes ; ! writes to terminal at all steps
auxiliary files = recip ;
log file = yes ;
exogenous x ;
rest endogenous ;
method = euler ;
steps = 2 ;
shock x = 20 ;
verbal description = test of y=-x ;
```

We suggest that you run this and look at the log file RECIP.LOG. The "dws = yes ;" statement in the Command file means that the values of X_L and Y_L are written in the LOG file at the start of each step (steps 1 and 2). The (postsim) writes show the post-simulation values of X_L and Y_L. Check from the log file that
X_L = 1 1 and Y_L = 0.9 at the start of step 2 (that is, after step 1), and
X_L = 1 2 and Y_L = 0.8181 in the postsim write (that is, after step 2).

Note that the Update statements are the formal connection between the levels values X_L,Y_L and the associated percent changes x,y respectively. You should interpret the update statement for Y_L as meaning

Updated Y_L = old Y_L * [1 + y/100]

This happens at each step. The calculation of the value of Y_L=0.8181 at the end of the second step shown in section 2.1 above is an example of this formula.

2.4 More Accurate Solve

You might like to solve the RECIP.TAB model more accurately. For example, change to "method = gragg ;" and "steps = 4 6 8 ;" in RECIP.CMF and run it.
You will find that y = –16.6667 as expected from our calculation above in section 2.

3 Numerical Example – Two 10% Increases in X

Here consider two (exogenous) increases of 10% in X, one after the other. [This is NOT how GEMPACK does a 20% shock to X as you have seen above). But it is gives a little more intuition about the calculations above.

First note that two increases of 10% compounds to more than 20% (actually to 21%). {First X goes from 1 to 1.1. Then X goes from 1.1 to 1.1*[1 + 10/100] = 1.21.}

But two decreases of 10% compounds to less than –20% (actually to –19%). {First X goes from 1 to 0.9. Then X goes from 0.9 to 0.9*[1 – 10/100] = 0.81.} Note that, in the second "step", 10% of the smaller value (0.9 after the first "step") is less than 10% of the original value.