Question:

In my model there are 2 endogenous variables X and Y which are functions of exogenous variables A and B. The variables X and Y have the particular property that percentage changes in them are equal whenever A is shocked but B is not shocked. I carried out a multi-step simulation in which A and B were both shocked, then ran the decomposition simulation to find out their contributions to changes in X and Y. I found that the A-subtotal results for X and Y are not equal, even though there are no B-shocks when the A-subtotal is calculated. Is this an error?

Background:

An example of this issue is given by the equation

C= GNP.APCGNP

which links final consumption (C) to GNP via the average propensity to consume out of GNP (APCGNP) in the MONASH model. The variable GNP, in turn, depends on production technology, among many other exogenous variables.

During a historical period, many exogenous variables may have changed, such as techonogies and tastes, which affect GNP and final consumption. In the decomposition simulation we would expect that the contribution of changes in technologies, for example, to GNP and C are the same, because the APCGNP would be assumed to be unchanged in that subtotal. However, there would be a small differences between the values for GNP and C in that subtotal, and *it is not an error* (as is explained below).

Using the notation in your question, GNP and C would be X and Y respectively, and technological coefficient A and APCGNP would be A and B respectively.

Answer:

The A-subtotal results for X and Y would be the same in a one-step simulation, but not in a multi-step simulation. The reason is that in GEMPACK, the A-subtotal is the total contribution of A to percentage changes in X and Y away from their *initial values* X_0 *and* Y_0. In a multi-step simulation, the variables are updated after each step. Therefore, even if percentage changes in X and Y caused by shocks to A are the same in a step, their changes relative to the initial values are not necessarily the same.

The total contributions of A to percentage changes in X and Y in a multi-step simulation are calculated according to the following formulas, which can be derived from the formulas in section 5.1 of Harrison *et al* (1999)[1]:

[1] In Harrison *et al* (1999), M is the number of simulations whereas in this paper M is the number of steps in a multi-step calculation.

$$C_{A\,to\,X} = \frac{1}{X_0}\sum_{s=1}^{M} X_{s-1} C_{A\,to\,X,s} \quad (1)$$

$$C_{A\,to\,Y} = \frac{1}{Y_0}\sum_{s=1}^{M} Y_{s-1} C_{A\,to\,Y,s} \quad (2)$$

where

$C_{A\,to\,X}$ and $C_{A\,to\,Y}$ are the total contribution of shocks to A to percentage changes in X and Y;

M is the number of steps in a multi-step simulation;

X_0 and Y_o are the initial values for X and Y at the beginning of the simulation;

X_{s-1} and Y_{s-1} are the values of X and Y at the start of step number s. They are the post-simulation values of step number s-1; and

$C_{A\,to\,X,s}$ and $C_{A\,to\,Y,s}$ are the contributions of A to X and Y at step number s.

In your model, $C_{A\,to\,X,s}$ and $C_{A\,to\,Y,s}$ are the same in all steps $s \in M$. But it is most likely that X_{s-1} and Y_{s-1} would not be the same if the effects of shocks of B to X and Y are different, because X_{s-1} and Y_{s-1} are the results of changes to X and Y when both A and B are shocked.

An example will make this clear. Suppose that we have the following model:

$$X = YB \qquad Y = A \quad (3)$$

where X and Y are endogenous, and A and B are exogenous.

The linearised version of the model is:

$$x = y + b \qquad y = a \quad (4)$$

From (4) we can see that if $b = 0 \Rightarrow x = y = a$

Assuming that the initial values of A and B are A_0=3 and B0=2. Now we would want to shock A and B with ΔA=1 and ΔB=1.

In the following analysis the linearised version of the model is used.

1) <u>If the simulation is run in one step</u>, the contribution of A to changes in X and Y can be calculated by shocking A only with the percentage change of a=1/3*100=33.33 percent. Then y=33.33 and x=33.33. And $C_{A\,to\,X}$ = $C_{A\,to\,Y}$ =33.33.

2) <u>If the simulation is run in two steps</u>. Assuming that the shocks are divided equally between the steps, both A and B would be shocked by the change of 0.5 unit in each step.

In the first step, A and B are shocked with a_1=16.67 percent (=0.5/3*100) and b_1=25 percent (=0.5/2*100). Then from (4) we have y_1=16.67 and x_1=16.67+25=41.67. The new values for X and Y are X_1=X0(1+x1/100)=8.5 and Y_1=Y0(1+y1/100)=3.5. The contributions of shocks of A to X and Y are both 16.67 percent.

In the second step, only A is shocked with a_2=14.29 percent. Then x_2=14.29 and y_2=14.29, which are the contribution of A to X and Y.

The total contribution of A to percentage changes in X is:

$$C_{A\,to\,X} = \frac{1}{X_0}(X_0.C_{A\,to\,X,1} + X_1.C_{A\,to\,X,2} = (3*16.67 + 8.5*14.29)/3 = 36.90$$; and

the total contribution of A to percentage changes in Y is:

$$C_{A\,to\,Y} = \frac{1}{Y_0}(Y_0.C_{A\,to\,X,1} + Y_1.C_{A\,to\,X,2} = (3*16.67 + 3.5*14.29)/3 = 33.33$$

which are different from each other.

A similar analysis can be conducted for simulations with more steps, and the results would still be that contributions of A to X and Y would likely be different in multi-step simulations.

References

W.J.Harrison, J.M.Horridge and K.R.Pearson (1991), "Decomposing Simulation Results with Respect to Exogenous Shocks", Centre of Policy Studies Preliminary Working Paper No. IP-73, May 1999, pp. 21 which can be downloaded from http://www.monash.edu.au/policy/elecpapr/ip-73.htm